2018
MATHEMATICS

Full Marks : 100

Pass Marks : 33

Time : Three hours

Attempt all Questions.

The figures in the right margin indicate full marks for the questions.

For Question Nos. 1-6, write the letter associated with the correct answer.

1. If $f:R\to R$ be given by $f(x)=(3-x^3)^{1/3}$, then *fof(x)* equals 1

   A. $(3-x^3)$

   B. $x^{1/3}$

   C. x^3

   D. x.

2. The principal value of is $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is 1

   A. π

   B. 2π

   C. $\frac{2\pi}{3}$

   D. $\frac{4\pi}{3}$.

3. If $P(A) = 0.6$, $P(B) = 0.9$ and $P\left(\frac{A}{B}\right) = 0.3$, then $P\left(\frac{B}{A}\right)$ equals 1

A. 0.27

B. 0.36

C. 0.45

D. 0.54.

4. The function $f(x) = (x+1)^3 \ (x-3)^3$ is increasing in the interval 1

A. $(-1, 3)$

B. $(-\infty, 1)$

C. $(1, \infty)$

D. $(-\infty, \infty)$.

5. $\int \frac{dx}{\sin^2 x \cos^2 x}$ equals 1

A. $\tan x \cot x + c$

B. $\tan x - \cot x + c$

C. $\tan x + \cot x + c$

D. $\cot x - \tan x + c$.

6. Distance between the planes $2x + 3y + 6z = 13$ and $2x + 3y + 6z = 6$ is 1

A. 1 unit

B. 2 units

C. 3 units

D. 7 units.

7. Find the identity element of the binary operation $*$ on $\boldsymbol{R}$ defined by $a*b=\frac{ab}{4}$, $\forall a,b \in \boldsymbol{R}$. 1

8. Is Rolle's Theorem applicable to the function $f(x) = \tan x$ in the interval $[0,\pi]$? 1

9. If $y = A\cos 2x + B\sin 2x$, find $\frac{d^2y}{dx^2}$ in terms of y. 1

10. Write down the slope of the normal to the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $(a\cos\theta,\ b\sin\theta)$. 1

11. Evaluate : $\int_0^1 \sqrt{1-x^2}\,dx$. 1

12. What is meant by the general solution of a differential equation ? 1

13. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+\hat{j}+2\hat{k}$, find a unit vector in the direction of $\vec{a}+\vec{b}$. 1

14. State Parallelogram Law of Vectors. 1

15. Vector equation of a line is $\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}-3\hat{j}-4\hat{k}\right)$. Write its Cartesian form. 1

16. Can the numbers $\frac{1}{\sqrt{2}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{6}}$ be the direction cosines of a line? Give reasons of your answer. 1

17. Show that the function $f : R \rightarrow R$ defined by $f(x) = 4x + 3,\ \forall x \in R$ is invertible and find the inverse of f. 3

18. If $= \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, prove that $A^3 = A^{-1}$. 3

19. Prove that $\int f(x).g(x)dx = f(x).\int g(x)dx - \int \{ f'(x).\int g(x)dx \} dx$. 3

20. Evaluate : $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\, dx$. 3

21. Find the differential equation of the family of curves $y = Ae^{5x} + Be^{3x}$, where A and B are arbitrary constants. 3

22. Find the mean and variance of the following distribution: 3

X	1	2	3	4
P(X)	0.4	0.3	0.2	0.1

23. Prove that $tan^{-1}x + tan^{-1}y = tan^{-1}\frac{x+y}{1-xy}, (xy < 1)$ and hence deduce that 4

(i) $tan^{-1}x - tan^{-1}y = tan^{-1}\frac{x-y}{1+xy}, (xy > -1)$

(ii) $2tan^{-1}x = tan^{-1}\frac{2x}{1-x^2}, (|x| < 1)$.

24. Prove that every square matrix can be expressed uniquely as the sum of a symmetric matrix and a skew – symmetric matrix. 4

25. Prove that the function f defined by 4

$$f(x)=\begin{cases} \frac{x^2}{a}-a & \text{, when } 0<x<a \\ 0 & \text{, when } x=a \\ a-\frac{a^3}{x^2} & \text{, when } x>a \end{cases}$$

is continuous and differentiable at the point $x = a$.

26. If $x^a y^b = (x+y)^{a+b}$, prove that $\frac{dy}{dx}=\frac{y}{x}$, provided $ay \neq bx$. 4

OR

If $x=a(\theta-\sin\theta)$ and $y=a(1-\cos\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{2}$.

27. Find, by integration, the area of the region bounded by the curves $y^2 = 4ax$ and $x^2 = 4ay$. 4

OR

Find, by integration, the area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum.

28. Write down the standard form of a linear differential equation of the first order and hence obtain the integrating factor of the equation. 4

29. Define cross product of two vectors and give the geometrical interpretation of the cross product of two vectors. If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, obtain the algebraic formula for $\vec{a}\times\vec{b}$. 6

30. State and prove Baye's Theorem. 6

31. Prove that : $\int_0^{\frac{\pi}{2}} \frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}\,dx = \frac{\pi^2}{16}$. 6

OR

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{\sqrt{a^2+b^2}}\sin\left(bx-\tan^{-1}\frac{b}{a}\right)+c.$$

32. Derive the vector equation of a line passing through a given point and parallel to a given vector and hence obtain the Cartesian equation of the line. 6

OR

Derive the vector equation of a plane passing through three given non collinear points and hence obtain the Cartesian equation of a plane in the intercept form.

33. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle α is $\frac{4}{27}\pi h^3\tan^2\alpha$. 6

OR

Find the point on the curve $y^2 = 4x$ which is nearest to the point (2, –8).

34. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find AB and hence solve the following system of linear equations : 6

$$x - y = 3$$
$$2x + 3y + 4z = 17$$
$$y + 2z = 7.$$

35. An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps D, E and F whose requirements are 4500 L, 3000 L and 3500 L respectively. The distances (in km) between the depots and the petrol pumps are given in the following table:

To / From	Distances (in km)		
	D	E	F
A	7	6	3
B	3	4	2

Assuming that the transportation cost of 10 litres of oil is Re. 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost? 6