22 Mth 10/16 (I)

2016

MATHEMATICS

Full Marks : 100

Pass Marks : 33

Time : Three Hours and *Fifteen Minutes

(*15 minutes are given as extra time for reading questions)

Attempt all Questions.

The figures in the right margin indicate full marks for the questions.

For Question Nos. ***1-6****, write the letter associated with the correct answer.*

1. If $f : R \to R$ be given by $f(x) = \left(3 - x^3\right)^{1/3}$, then $fof(x)$ equals :

   (A) $3 - x^3$

   (B) $x^{1/3}$

   (C) x^3

   (D) x 1

2. If $tan^{-1} x + tan^{-1} y = \frac{\pi}{4}$ and $xy < 1$, then the value of $x + y + xy$ is :

   (A) -1

   (B) 1

   (C) 0

   (D) 2 1

3. The length of the perpendicular from the point whose position vector is $\vec{a}$ upon the plane $\vec{r}.\vec{n} = q$ is :

   (A) $\frac{|q - \vec{a}.\vec{n}|}{|\vec{n}|}$

   (B) $\frac{|\vec{n}|}{|q - \vec{a}.\vec{n}|}$

   (C) $\frac{|q - \vec{a}.\vec{n}|}{q}$

   (D) $|q - \vec{a}.\vec{n}|$ 1

4. Projection vector of $\vec{a}$ on $\vec{b}$ is :

   (A) $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

   (B) $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$

   (C) $\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\vec{b}$

   (D) $\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}\vec{a}$ 1

5. The rate of change of the area of a circle with respect to its radius at $r = 6\,cm$ is:

(A) $8\pi\ cm^2/cm$

(B) $10\pi\ cm^2/cm$

(C) $11\pi\ cm^2/cm$

(D) $12\pi\ cm^2/cm$ 1

6. The degree of the differential equation

$\left(\frac{d^3y}{dx^3}\right)^2 - \left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + y = 0$ is:

(A) 4

(B) 3

(C) 2

(D) 1 1

7. What is meant by an equivalence relation ? 1

8. Write down the equation of the tangent to the curve $y = f(x)$ at the point (x_1, y_1). 1

9. State the range of $cosec^{-1}x$. 1

10. For any square matrix A, prove that AA' is a symmetric matrix. 1

11. Give geometrical interpretation of *Lagrange's Mean Value Theorem.* 1

12. If $y = a\sin 3x - b\cos 3x$, write down the value of $\frac{d^2y}{dx^2}$ in terms of y. 1

13. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$, what is the value of $\vec{a}.(\vec{b}\times\vec{c})$? 1

14. Write down the expression for the distance between the parallel lines $\vec{r} = \vec{a_1} + \lambda\vec{b}$ and $\vec{r} = \vec{a_2} + \mu\vec{b}$. 1

15. If α, β, γ be the angles made by a line with the coordinate axes, prove that $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$ 1

16. If $P(B) = 0{\cdot}8$ and $P(A|B) = 0{\cdot}4$, find $P(A \cap B)$. 1

17. Show that the function $f : R \to R$ defined by $f(x) = 3 - 4x\ \forall\ x \in R$ is invertible and find the inverse of f. 3

18. Prove that $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$. 3

19. If A is non-singular square matrix of order 3, prove that

$$A^{-1} = \frac{1}{|A|}(adj\ A)$$ 3

20. Using properties of determinants, prove that 3

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

21. What is meant by a homogeneous differential equation ? Describe how it can be reduced to a form in which the variables are separable. 3

22. What is meant by a random variable ? Define its probability distribution and mean. 3

23. If the function f defined by

$$f(x)=\begin{cases} \dfrac{\sin(a+1)x+\sin x}{x}, & \text{if } x<0 \\ c, & \text{if } x=0 \\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}}, & \text{if } x>0 \end{cases}$$

is continuous at the point $x=0$, find the values of a, b and c. 4

Or

If $\cos y = x\cos(a+y)$, with $\cos a \neq \pm 1$, prove that $\dfrac{dy}{dx}=\dfrac{\cos^2(a+y)}{\sin a}$.

24. Prove that

$$\int\sqrt{a^2-x^2}\,dx=\frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\sin^{-1}\frac{x}{a}\right]+C,\ (|x|\le|a|).$$ 4

25. Prove that : $\int e^{ax}\sin bx\,dx=\dfrac{e^{ax}}{a^2+b^2}(a\sin bx-b\cos bx)+C$. 4

26. Find, by integration, the area of the region bounded by the triangle whose vertices are (1,2), (2,4) and (3,1). 4

Or

Find, by integration, the area of the region bounded by the parabolas $x^2=y$ and $y^2=x$.

27. Solve : $\cos^2 x\dfrac{dy}{dx}+y=\tan x$, given that $y=1$ when $x=0$. 4

28. If a function f is differentiable at a point, prove that it is also continuous at that point. 4

29. If $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, prove that $A^3-6A^2+9A-4I=0$ and hence find A^{-1}. 6

30. For a given curved surface of right circular cone whose volume is maximum, show that the semi-vertical angle is $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$. 6

Or

Show that a closed right circular cylinder of given surface area and maximum volume is such that its height is equal to the diameter of its base.

31. If $\vec{a}, \vec{b}$ and $\vec{c}$ be the position vectors of the vertices of a triangle, find the position vector of the centroid of the triangle, and hence deduce that the medians of a triangle are concurrent. 6

Or

In a triangle *ABC*, if the vectors $\overrightarrow{BC}, \overrightarrow{CA}$, and $\overrightarrow{AB}$ be represented by $\vec{a}, \vec{b}$ and $\vec{c}$ respectively, show that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$, and hence deduce that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

32. Derive the vector equation of a line passing through a given point and parallel to a given vector, and hence obtain the *Cartesian* equation of the line. 6

Or

Derive the vector equation of a plane passing through a given point and perpendicular to a given vector, and hence obtain the *Cartesian* equation of the plane.

33. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$. 6

34. A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on machine C. The profit obtained by selling one chair is Rs. 30 while by selling one table the profit is Rs. 60. The total time available per week on machine A is 70 hours, on machine B is 40 hours, and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit ? Formulate the problem as a L.P.P. and also solve it graphically. 6

35. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. 6